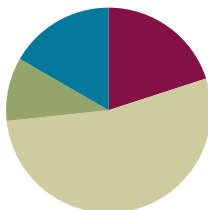# Lesson 4

Objective: Convert numerical expressions into unit form as a mental strategy for multi-digit multiplication.

Suggested Lesson Structure

■ Fluency Practice   (12 minutes)
■ Application Problem   (6 minutes)
□ Concept Development   (32 minutes)
■ Student Debrief   (10 minutes)

    Total Time   **(60 minutes)**

Fluency Practice (12 minutes)

▪ Estimate Products **5.NBT.6**   (4 minutes)
▪ Decompose Multiplication Sentences **3.OA.5**   (4 minutes)
▪ Write the Value of the Expression **5.OA.1**   (4 minutes)

Estimate Products (4 minutes)

Materials: (S) Personal white board

   T: (Write 409 × 21 ≈ ____ × ____ = ____.) On your personal white board, write the multiplication sentence rounding each factor to arrive at a reasonable estimate of the product.
   S: (Write 409 × 21 ≈ 400 × 20 = 8,000.)

Repeat the process and procedure for 287 × 64; 3,875 × 92; and 6,130 × 37.

Decompose Multiplication Sentences (4 minutes)

Materials: (S) Personal white board

   T: (Write 12 × 3 = ____.) Write the multiplication sentence.
   S: (Write.)
   T: (Write (8 × 3) + (____ × 3) = ____ below 12 × 3 = ____.) 12 is the same as 8 and what number?
   S: 4.
   T: (Write (8 × 3) + (4 × 3) = ____. Below it, write 24 + ____ = ____.) Fill in the blanks.
   S: (Write 12 × 3 = 36. Below it, they write (8 × 3) + (4 × 3) = 36. Below that line, they write 24 + 12 = 36.)

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Repeat using the following possible sequence: 14 × 4, 13 × 3, and 15 × 6, changing the missing numbers that students need to fill in.

Write the Value of the Expression (4 minutes)

Materials: (S) Personal white board

> T: (On the board, write 11 × (15 + 5).) Write the expression as a single multiplication sentence without parentheses.
>
> S: (Write 11 × 20 = 220.)

Repeat the process for (41 − 11) × 12, (75 + 25) × 38, and (20 × 2) + (6 × 2).

Application Problem (6 minutes)

Jaxon earned $39 raking leaves. His brother, Dayawn, earned 7 times as much waiting on tables. Write a numerical expression to show Dayawn's earnings. How much money did Dayawn earn?

Note: This problem is simple enough that students can solve it using pencil and paper prior to this lesson. Allow students to share their approach to solving. However, in the Debrief, students are asked to return to the Application Problem and solve this problem again applying a new mental strategy to evaluate.
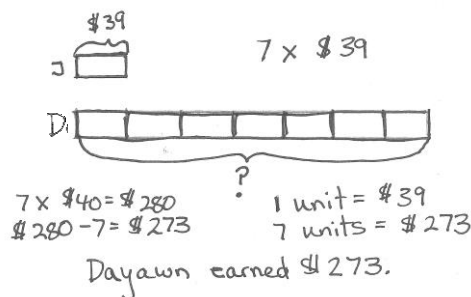
Concept Development (32 minutes)

Materials: (S) Personal white board

Problems 1–2

8 × 31

8 × 29

> T: (Show 8 × 31 on the board.) What does this expression mean when I designate 31 as the unit?
>
> S: Add thirty-one 8 times. → 8 times as much as thirty-one.
>
> T: What does it mean when I designate 8 as the unit?
>
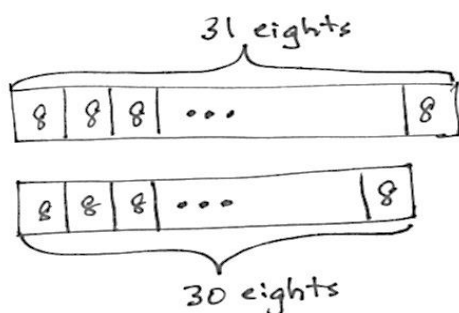> S: Add eight 31 times. → 31 times as much as eight.
>
> T: Does our choice of unit change the product of the two factors?
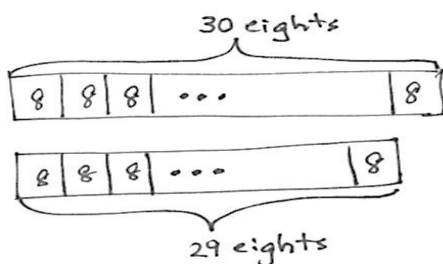
> **NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:**
>
> This lesson requires students to work mentally with two-digit and three-digit numbers. If basic multiplication facts are not yet mastered, be prepared to adjust numbers in calculations to suit the learner's level. A good time to review mental math strategies is during Sprints and fluency activities. Spending time working on basic facts (with flash cards, computer games, etc.) may be necessary prior to this lesson.

S: No.

T: Why not? What property allows for this?

S: The commutative property (any-order property) says that the order of the factors doesn't matter. The product will be the same.

T: Let's designate 8 as the unit. I've drawn diagrams of 8 × 31 and 8 × 30.

T: Use the diagrams to consider how 8 × 30 helps us to solve 8 × 31 when we designate eight as the unit (point to the diagram) and the other factor as the number of units, 31 and 30. (Run your finger down the length of each bar.) Turn and talk.

> **NOTES ON MULTIPLE MEANS OF REPRESENTATION:**
>
> Possibly challenge students to (a) solve the problem designating 31 as the unit and (b) think of other ways to decompose 31 units of 8.
>
> T: Could we have decomposed 31 eights in another way? Turn and talk.
>
> S: (Students share.)
>
> T: Yes! 31 eights is also equivalent to 20 eights plus 11 eights. Would this way of decomposing 31 change the product of 8 x 31?
>
> S: No. It would be the same because 20 eights is 160 and 11 eights is 88, which is the same as 160 + 88, which is 248.

S: 31 eights is the same as 30 eights plus 1 eight. → 30 eights is 240 and one more eight makes 248. → 30 eights is easy, 240. 240 + 8 = 248.

T: How many more eights are in the first bar than in the second bar?

S: 1 more eight.

T: Let's record our thinking. (Write 31 eights = 30 eights + 1 eight. 31 × 8 = (30 × 8) + (1 × 8).)

T: What is the value of 30 eights and 1 more eight? Say it in an addition sentence that corresponds to our last equation. (Point to (30 × 8) + (1 × 8).)

S: 240 + 8 = 248.

T: 31 times 8 is…?

S: 248.

T: (Show 8 × 29 on the board.) What does this expression mean when we designate eight as the unit?

S: Add 8 twenty-nine times. → Add 8 over and over 29 times.

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T: How does 8 × 30 helps us to solve 8 × 29? Turn and talk.

S: (Discuss.)

T: I heard Jackie say that 30 eights minus 1 eight is equal to 29 eights. (Write 30 eights – 1 eight = 29 eights. 29 × 8. (30 × 8) – (1 × 8) = 8 × 29.)

T: What is the value of 30 eights minus 1 eight?

S: 232.

T: Could we have decomposed 29 eights in another way to help us evaluate the expression mentally? Turn and talk.

S: (Share.)

T: Yes! 29 eights is also equivalent to 20 eights plus 9 eights. Would this way of decomposing 29 change the product of 8 × 29?

S: No.

T: Why not?

S: Because it is still 29 eights even though we found 20 eights then 9 eights. → 20 eights = 160 and 9 eights is 72. That's still 232.

Problems 3–4

49 × 20

20 × 51

T: (Write 49 × 20.) To solve this mentally using today's strategy, first determine which factor will be designated as the unit. Which is easier to work with: 49 twenties or 20 forty-nines? Turn and talk.

S: It is easier to think of 20 as the unit because then we can say 40 twenties and 9 twenties. → It's easier to think of twenty as the unit because it is 1 less than 50 twenties. (Students might also share why 49 is easier.)

T: Let's agree to designate 20 as the unit. Go ahead and find the value of the expression using today's unit form strategy. Use a tape diagram if you so choose.
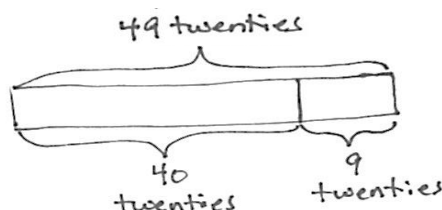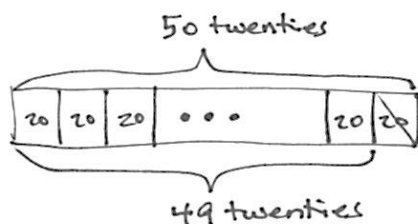
S: (Work and share.)

T: What is the value of 49 × 20?

S: 980.

T: Work with a partner to create an equivalent expression that you can use to help you solve 20 × 51 mentally. Write the equivalent expression and its value on your personal white board. As before, you may draw a tape diagram if you choose.

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S: (Work and share.)

T: (Circulate and assess for understanding. Be receptive to any valid mental approach.)

Problems 5–6

101 × 12

12 × 98

T: Work independently to evaluate these two expressions mentally. (Write 12 × 98 and 12 × 101 on the board.) Compare your work with a neighbor when you're finished. Draw tape diagrams if you choose.

S: (Work.)

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Convert numerical expressions into unit form as a mental strategy for multi-digit multiplication.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- What mental math strategy did you learn today? (Unit form.) Choose a problem in the Problem Set to support your answer.
- How did the Application Problem connect to today's lesson? Which factor did you decide to designate as the unit?
- In Problem 1(b) the first two possible expressions are very similar. How did you decide which one was not equivalent?

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- Look at Problem 2. How did the think prompts help to guide you as you evaluated these expressions? Turn and talk.

- What was different about the think prompts in Problem 2 and Problem 3? (Problem 2 prompts give the units but not the number of units. Problem 3 prompts give the number of units but not the name of the units.)

- Explain to your partner how to solve Problem 5(a). (Some students may have thought 101 × 15 = (101 × 10) + (101 × 5), while others may see that 101 × 15 = (100 × 15) + (1 × 15). Both are acceptable.)

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

Name _____   Date _____

1. Circle each expression that is not equivalent to the expression in **bold**.

    a. **16 × 29**

    29 sixteens   16 × (30 − 1)   (15 − 1) × 29   (10 × 29) − (6 × 29)

    b. **38 × 45**

    (38 + 40) × (38 + 5)   (38 × 40) + (38 × 5)   45 × (40 + 2)   45 thirty-eights

    c. **74 × 59**

    74 × (50 + 9)   74 × (60 − 1)   (74 × 5) + (74 × 9)   59 seventy-fours

2. Solve using mental math. Draw a tape diagram and fill in the blanks to show your thinking. The first one is partially done for you.

    a. 19 × 25 = _____ twenty-fives

    | 25 | 25 | 25 | ... | 25 | 25 (crossed out) |
    | 1 | 2 | 3 | ... | 19 | 20 |

    Think: 20 twenty-fives − 1 twenty-five.

    = (_____ × 25) − (_____ × 25)

    = _____ − _____

    = _____

    b. 24 × 11 = _____ twenty-fours

    Think: _____ twenty fours + _____ twenty four

    = (_____ × 24) + (_____ × 24)

    = _____ + _____

    = _____

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c. 79 × 14 = _____ fourteens

Think: _____ fourteens − 1 fourteen

= (_____ × 14) − (_____ × 14)

= _____ − _____

= _____

d. 21 × 75 = _____ seventy-fives

Think: _____ seventy-fives + _____ seventy-five

= (_____ × 75) + (_____ × 75)

= _____ + _____

= _____

3. Define the unit in word form and complete the sequence of problems as was done in the lesson.

a. 19 × 15 = 19 _____

Think: 20 _____ − 1 _____

= (20 × _____) − (1 × _____)

= _____ − _____

= _____

b. 14 × 15 = 14 _____

Think: 10 _____ + 4 _____

= (10 × _____) + (4 × _____)

= _____ + _____

= _____

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c.   25 × 12 = 12 _____   d.   18 × 17 = 18 _____

Think: 10 _____ + 2 _____   Think: 20 _____ − 2 _____

= (10 × _____) + (2 × _____)   = (20 × _____) − (2 × _____)

= _____ + _____   = _____ − _____

= _____   = _____

4.   How can 14 × 50 help you find 14 × 49?

5.   Solve mentally.
   a.   101 × 15 = _____   b.   18 × 99 = _____

6.   Saleem says 45 × 32 is the same as (45 × 3) + (45 × 2). Explain Saleem's error using words, numbers, and/or pictures.

7.   Juan delivers 174 newspapers every day. Edward delivers 126 more newspapers each day than Juan.
   a.   Write an expression to show how many newspapers Edward will deliver in 29 days.

   b.   Use mental math to solve. Show your thinking.

Name _____   Date _____

1. Solve using mental math. Draw a tape diagram and fill in the blanks to show your thinking.

a. 49 × 11 = _____ elevens	b. 25 × 13 = _____ twenty-fives

Think: 50 elevens – 1 eleven

   = (_____ × 11) – (_____ × 11)

   = _____ – _____

   = _____

Think: _____ twenty-fives + _____ twenty-fives

   = (_____ × 25) + (_____ × 25)

   = _____ + _____

   = _____

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Name _____ Date _____

1. Circle each expression that is not equivalent to the expression in **bold**.

   a. **37 × 19**

   37 nineteens   (30 × 19) − (7 × 29)   37 × (20 − 1)   (40 − 2) × 19

   b. **26 × 35**

   35 twenty-sixes   (26 + 30) × (26 + 5)   (26 × 30) + (26 × 5)   35 × (20 + 60)

   c. **34 × 89**

   34 × (80 + 9)   (34 × 8) + (34 × 9)   34 × (90 − 1)   89 thirty-fours

2. Solve using mental math. Draw a tape diagram and fill in the blanks to show your thinking. The first one is partially done for you.

a. 19 × 50 = _____ fifties	b. 11 × 26 = _____ twenty-sixes
Think: 20 fifties − 1 fifty	Think: _____ twenty-sixes + _____ twenty-sixes
= (_____ × 50) − (_____ × 50)	= (_____ × 26) + (_____ × 26)
= _____ − _____	= _____ + _____
= _____	= _____

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c.   49 × 12 = _____ twelves

Think: _____ twelves − 1 twelve

    = (_____ × 12) − (_____ × 12)

    = _____ − _____

    = _____

d.   12 × 25 = _____ twenty-fives

Think: _____ twenty-fives + _____ twenty-fives

    = (_____ × 25) + (_____ × 25)

    = _____ + _____

    = _____

3.   Define the unit in word form and complete the sequence of problems as was done in the lesson.

a.   29 × 12 = 29 _____

Think: 30 _____ − 1 _____

    = (30 × _____) − (1 × _____)

    = _____ − _____

    = _____

b.   11 × 31 = 31 _____

Think: 30 _____ + 1 _____

    = (30 × _____) + (1 × _____)

    = _____ + _____

    = _____

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Lesson 4:   Convert numerical expressions into unit form as a mental strategy for multi-digit multiplication.

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c.   19 × 11 = 19 _____

d.   50 × 13 = 13 _____

Think: 20 _____ − 1 _____

= (20 × _____) − (1 × _____)

= _____ − _____

= _____

Think: 10 _____ + 3 _____

= (10 × _____) + (3 × _____)

= _____ + _____

= _____

4.   How can 12 × 50 help you find 12 × 49?

5.   Solve mentally.

a.   16 × 99 = _____

b.   20 × 101 = _____

6.   Joy is helping her father to build a rectangular deck that measures 14 ft by 19 ft. Find the area of the deck using a mental strategy. Explain your thinking.

7.   The Lason School turns 101 years old in June. In order to celebrate, they ask each of the 23 classes to collect 101 items and make a collage. How many total items will be in the collage? Use mental math to solve. Explain your thinking.

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