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# Relationships Between Quantities and Reasoning with Equations and Their Graphs

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[1] Each lesson is ONE day, and ONE day is considered a 45-minute period.

*Topics A through D (assessment 2 days, return and remediation or further applications 4 days)*

Algebra I • Module 1

# Relationships Between Quantities and Reasoning with Equations and Their Graphs

## OVERVIEW

By the end of Grade 8, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students are introduced to non-linear equations and their graphs. Students formalize their understanding of equivalent algebraic expressions and begin their study of polynomial expressions. Further, they learn that there are some actions that, when applied to the expressions on both sides of an equal sign, will not result in an equation with the same solution set as the original equation. Finally, they encounter problems that induce the full modeling cycle, as it is described in the Common Core Learning Standards for Mathematics.

In Topic A, students explore the main functions that they will work with in Algebra I: linear, quadratic, and exponential. The goal is to introduce students to these functions by having them make graphs of situations (usually based upon time) in which the functions naturally arise (**A-CED.A.2**). As they graph, they reason abstractly and quantitatively as well as choose and interpret units to solve problems related to the graphs they create (**N-Q.A.1**, **N-Q.A.2**, **N-Q.A.3**).

In middle school, students applied the properties of operations to add, subtract, factor, and expand expressions (**6.EE.A.3**, **6.EE.A.4**, **7.EE.A.1**, **8.EE.A.1**). Now, in Topic B, students use the structure of expressions to define what it means for two algebraic expressions to be equivalent. In doing so, they discern that the commutative, associative, and distributive properties help link each of the expressions in the collection together, even if the expressions look very different themselves (**A-SSE.A.2**). They learn the definition of a polynomial expression and build fluency in identifying and generating polynomial expressions as well as adding, subtracting, and multiplying polynomial expressions (**A-APR.A.1**). The Mid-Module Assessment follows Topic B.

Throughout middle school, students practice the process of solving linear equations (**6.EE.B.5**, **6.EE.B.7**, **7.EE.B.4**, **8.EE.C.7**) and systems of linear equations (**8.EE.C.8**). Now, in Topic C, instead of just solving equations, they formalize descriptions of what they learned before (variable, solution sets, etc.) and are able to explain, justify, and evaluate their reasoning as they strategize methods for solving linear and non-linear equations (**A-REI.A.1**, **A-REI.B.3**, **A-CED.A.4**). Students take their experience solving systems of linear equations further as they prove the validity of the addition method, learn a formal definition for the graph of an equation and use it to explain the reasoning of solving systems graphically, and represent the solution to systems of linear inequalities graphically (**A-CED.A.3**, **A-REI.C.5**, **A-REI.C.6**, **A-REI.D.10**, **A-REI.D.12**).

In Topic D, students are formally introduced to the modeling cycle (see page 61 of the CCLS) through problems that can be solved by creating equations and inequalities in one variable, systems of equations, and graphing (**N-Q.A.1**, **A-SSE.A.1**, **A-CED.A.1**, **A-CED.A.2**, **A-REI.B.3**). The End-of-Module Assessment follows Topic D.

**COMMON CORE**　　Module 1:　Relationships Between Quantities and Reasoning with Equations and Their Graphs　　**engage**^ny　　**3**

Date:　10/8/14~~5/30/14~~

# Focus Standards

## Reason quantitatively and use units to solve problems.

**N-Q.A.1**    Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.★

**N-Q.A.2**[2]    Define appropriate quantities for the purpose of descriptive modeling.★

**N-Q.A.3**    Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★

## Interpret the structure of expressions.

**A-SSE.A.1**    Interpret expressions that represent a quantity in terms of its context.★

  a. Interpret parts of an expression, such as terms, factors, and coefficients.

  b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.*

**A-SSE.A.2**    Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

## Perform arithmetic operations on polynomials.

**A-APR.A.1**    Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

## Create equations that describe numbers or relationships.

**A-CED.A.1**[3]    Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★

**A-CED.A.2**    Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

**A-CED.A.3**    Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*★

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[2] This standard will be assessed in Algebra I by ensuring that some modeling tasks (involving Algebra I content or securely held content from Grades 6-8) require the student to create a quantity of interest in the situation being described.

[3] In Algebra I, tasks are limited to linear, quadratic, or exponential equations with integer exponents.

**A-CED.A.4**   Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*★

### Understand solving equations as a process of reasoning and explain the reasoning.

**A-REI.A.1**   Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

### Solve equations and inequalities in one variable.

**A-REI.B.3**   Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

### Solve systems of equations.

**A-REI.C.5**   Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

**A-REI.6**[4]   Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

### Represent and solve equations and inequalities graphically.

**A-REI.D.10**   Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

**A-REI.D.12**   Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

# Foundational Standards

### Apply and extend previous understandings of numbers to the system of rational numbers.

**6.NS.C.7**   Understand ordering and absolute value of rational numbers.

  a.   Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret −3 > −7 as a statement that −3 is located to the right of −7 on a number line oriented from left to right.*

  b.   Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write −3°C > −7°C to express the fact that −3°C is warmer than −7°C.*

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[4] Tasks have a real-world context. In Algebra I, tasks have hallmarks of modeling as a mathematical practice (less defined tasks, more of the modeling cycle, etc.).

## Apply and extend previous understandings of arithmetic to algebraic expressions.

**6.EE.A.3**   Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

**6.EE.A.4**   Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

## Reason about and solve one-variable equations and inequalities.

**6.EE.B.5**   Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

**6.EE.B.6**   Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or depending on the purpose at hand, any number in a specified set.

**6.EE.B.7**   Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which $p$, $q$ and $x$ are all nonnegative rational numbers.

**6.EE.B.8**   Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

## Use properties of operations to generate equivalent expressions.

**7.EE.A.1**   Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

**7.EE.A.2**   Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*

## Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

**7.EE.B.3**   Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

**7.EE.B.4**    Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

     a.   Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where $p$, $q$, and $r$ are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

     b.   Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where $p$, $q$, and $r$ are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

## Work with radicals and integer exponents.

**8.EE.A.1**    Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.*

**8.EE.A.2**    Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where $p$ is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

## Analyze and solve linear equations and pairs of simultaneous linear equations.

**8.EE.C.7**    Solve linear equations in one variable.

     a.   Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where $a$ and $b$ are different numbers).

     b.   Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

**8.EE.C.8**    Analyze and solve pairs of simultaneous linear equations.

     a.   Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

     b.   Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*

     c.   Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

# Focus Standards for Mathematical Practice

**MP.1**   **Make sense of problems and persevere in solving them.**  Students are presented with problems that require them to try special cases and simpler forms of the original problem to gain better understanding of the problem.

**MP.2**   **Reason abstractly and quantitatively.**  Students analyze graphs of non-constant rate measurements and reason from the shape of the graphs to infer what quantities are being displayed and consider possible units to represent those quantities.

**MP.3**   **Construct viable arguments and critique the reasoning of others.**  Students reason about solving equations using "if-then" moves based on equivalent expressions and properties of equality and inequality.  They analyze when an "if-then" move is not reversible.

**MP.4**   **Model with mathematics.**  Students have numerous opportunities in this module to solve problems arising in everyday life, society, and the workplace from modeling bacteria growth to understanding the federal progressive income tax system.

**MP.6**   **Attend to precision.**  Students formalize descriptions of what they learned before (variables, solution sets, numerical expressions, algebraic expressions, etc.) as they build equivalent expressions and solve equations.  Students analyze solution sets of equations to determine processes (e.g., squaring both sides of an equation) that might lead to a solution set that differs from that of the original equation.

**MP.7**   **Look for and make use of structure.**  Students reason with and about collections of equivalent expressions to see how all the expressions in the collection are linked together through the properties of operations.  They discern patterns in sequences of solving equation problems that reveal structures in the equations themselves:  $2x + 4 = 10$, $2(x - 3) + 4 = 10$, $2(3x - 4) + 4 = 10$, etc.

**MP.8**   **Look for and express regularity in repeated reasoning.**  After solving many linear equations in one variable (e.g., $3x + 5 = 8x - 17$), students look for general methods for solving a generic linear equation in one variable by replacing the numbers with letters:  $ax + b = cx + d$.  They have opportunities to pay close attention to calculations involving the properties of operations, properties of equality, and properties of inequality as they find equivalent expressions and solve equations, noting common ways to solve different types of equations.

# Terminology

## New or Recently Introduced Terms

- **Piecewise-Linear Function** (Given a finite number of non-overlapping intervals on the real number line, a *(real) piecewise-linear function* is a function from the union of the intervals to the set of real numbers such that the function is defined by (possibly different) linear functions on each interval.)
- **Numerical Symbol** (A *numerical symbol* is a symbol that represents a specific number.)

- **Variable Symbol** (A *variable symbol* is a symbol that is a placeholder for a number. It is possible that a question may restrict the type of number that a placeholder might permit, maybe integers only or a positive real number, for instance.)

- **Numerical Expression** (A *numerical expression* is an algebraic expression that contains only numerical symbols (no variable symbols) and that evaluates to a single number.)

- **Algebraic Expression** (An *algebraic expression* is either: (1) a numerical symbol or a variable symbol or (2) the result of placing previously generated algebraic expressions into the two blanks of one of the four operators $((\_\_) + (\_\_), \ (\_\_) - (\_\_), \ (\_\_) \times (\_\_), \ (\_\_) \div (\_\_))$ or into the base blank of an exponentiation with an exponent that is a rational number.)

- **Equivalent Numerical Expressions** (Two numerical expressions are *equivalent* if they evaluate to the same number.)

- **Equivalent Algebraic Expressions** (Two algebraic expressions are *equivalent* if we can convert one expression into the other by repeatedly applying the commutative, associative, and distributive properties and the properties of rational exponents to components of the first expression.)

- **Polynomial Expression** (A *polynomial expression* is either: (1) a numerical expression or a variable symbol or (2) the result of placing two previously generated polynomial expressions into the blanks of the addition operator $(\_\_ + \_\_)$ or the multiplication operator $(\_\_ \times \_\_)$.)

- **Monomial** (A *monomial* is a polynomial expression generated using only the multiplication operator $(\_\_ \times \_\_)$. Monomials are products whose factors are numerical expressions or variable symbols.)

- **Degree of a Monomial** (The *degree of a non-zero monomial* is the sum of the exponents of the variable symbols that appear in the monomial.)

- **Standard Form of a Polynomial Expression in One Variable** (A polynomial expression with one variable symbol $x$ is in *standard form* if it is expressed as $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$, where $n$ is a non-negative integer, and $a_0, a_1, a_2, \ldots, a_n$ are constant coefficients with $a_n \neq 0$. A polynomial expression in $x$ that is in standard form is often called a *polynomial in $x$*.)

- **Degree of a Polynomial in Standard Form** (The *degree of a polynomial in standard form* is the highest degree of the terms in the polynomial, namely $n$.)

- **Leading Term and Leading Coefficient of a Polynomial in Standard Form** (The term $a_n x^n$ is called the *leading term,* and $a_n$ is called the *leading coefficient.*)

- **Constant Term of a Polynomial in Standard Form** (The *constant term* is the value of the numerical expression found by substituting 0 into all the variable symbols of the polynomial, namely $a_0$.)

- **Solution** (A *solution* to an equation with one variable is a number in the domain of the variable that, when substituted for all instances of the variable in both expressions, makes the equation a true number sentence.)

- **Solution Set** (The set of solutions of an equation is called its *solution set.*)

- **Graph of an Equation in Two Variables** (The set of all points in the coordinate plane that are solutions to an equation in two variables is called the *graph of the equation.*)

- **Zero Product Property** (The Zero Product Property states that given real numbers, $a$ and $b$, if $a \cdot b = 0$ then either $a = 0$ or $b = 0$, or both $a$ and $b = 0$ .)

## Familiar Terms and Symbols[5]

- Equation
- Identity
- Inequality
- System of Equations
- Properties of Equality
- Properties of Inequality
- Solve
- Linear Function
- Formula
- Term

# Suggested Tools and Representations

- Coordinate Plane
- Equations and Inequalities

# Assessment Summary

| Assessment Type | Administered | Format | Standards Addressed |
|---|---|---|---|
| Mid-Module Assessment Task | After Topic B | Constructed response with rubric | N-Q.A.1, N-Q.A.2, N-Q.A.3, A-APR.A.1, A-SSE.A.2 |
| End-of-Module Assessment Task | After Topic D | Constructed response with rubric | N-Q.A.1, A-SSE.A.1, A-SSE.A.2, A-APR.A.1, A-CED.A.1, A-CED.A.2, A-CED.A.3, A-CED.A.4, A-REI.A.1, A-REI.B.3, A-REI.C.5, A-REI.C.6, A-REI.D.10, A-REI.D.12 |

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[5] These are terms and symbols students have seen previously.